Teaching about the meaning of FRACTIONS: a conceptual and constructivist approach

Most of what follows is adapted from the Visual Mathematics Curriculum, written by Linda Cooper Foreman and Albert B. Bennett, Jr. and published by the Math Learning Center. Their materials are readily available. Check out the Math Learning Center's website @ www.mathlearningcenter.org

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What you'll need:

- ✓ One 12-egg carton OR ruler per person (include yourself). At the very minimum you'll need at least 3 egg cartons OR 3 rulers total.
- ✓ Copies of egg carton paper OR copies of "foot" grid paper if you're using rulers
- ✓ 6 pieces of string or yarn per person about a foot or so long OR 12 flat toothpicks if you're using rulers
- ✓ Something that you can use to fill sections of an egg carton or ruler (something like beans, marshmallows or square inch tiles, etc.), 12 per person and at least 36 total
- ✓ A couple of dollars in change with a variety of coins (include some pennies)
- ✓ Graph paper (1/4" would be good – or a bit bigger)

Note: The directions that follow are given for the egg carton fraction models, but you could easily do this with feet (as in rulers) and probably other things as well. Substitute the words "egg carton" for whatever other object you're choosing to use.

The point with these activities is NOT for you to be teaching the learners a bunch of rules about fractions. The point IS for learners to be given an opportunity to explore some key fraction concepts and to create knowledge (and real understanding) for themselves. Give them time to think things through, grapple with concepts, make mistakes and ponder their mistakes. If you're working with more than one learner, be sure to give plenty of time for people to share their thinking and their models. If you're working with one learner, you'll both want to do some thinking out loud.

1 Start by having a conversation with learners (or ask them to write) about when they use fractions in their life, what they already think they know about fractions, and anything they they'd like to know or understand.

2 Everyone (including you) will need one egg carton, 6 pieces of string and 12 items that can be used to fill the egg slots (beans, hard boiled eggs, marshmallows, whatever…)

3 Ask the learners to show how they might use the materials to demonstrate how they think of the meaning of "one half." Explain that there's no one right way – it's all open. The teacher/tutor does it too. Discuss and share your models.

4 The teacher/tutor makes a model of two-thirds, carefully using the yarn to split the carton into 3 equal parts and filling two of them. Then, write the number "2/3" and ask the learners how they think that the symbol "2/3" relates to your model. Then ask the learners to find some other ways that they could subdivide the carton to show 2/3 of a carton. (If you're using a ruler, move on to step 5 below).

5 Do another model, say of ¾ or 2/6, if you think your learners could use another example in order to figure out how the model relates to the symbol. Again, the tutor writes the symbol – don't just say the name of the fraction.

Ask learners what they think the bottom and top numbers of the fractions represent. You want the learners to get to the point where they can see for themselves that the bottom # tells how many equal parts the whole carton has been subdivided into and the top # is the number of those parts under consideration. (This is the part-to-whole concept of a fraction). You may also want to simply introduce the terms "numerator" and "denominator" at this point. Explain that right now what you're exploring together is the Part-to-Whole Concept of a fraction.

6 Ask the learners to find ways to subdivide the carton to show ¾ (if you didn't do it above) and 5/6, and 5/12 – each time now using the string to really clearly show the subdivisions and filling the appropriate number of sections. Take time for this. Ask learners to explain their thinking – it'll help them learn, and it'll help you get a better understanding of what they do and don't understand. You could make some models too, perhaps demonstrating some different ways to model the same fractions they've also modeled. Throw in additional fractions for each of you to model if you think that this would help the learners prepare for step 7 below.

7 Take out the egg carton grid paper and ask the learners to draw as many models of fractions of a dozen they can come up with. The teacher/tutor should do this, too. ***Remind the learners to be sure to draw the string lines/subdivisions on each model, fill the appropriate # of subsections, and write the fraction symbol that the model represents – so that each model is SUPER clear – everyone should be able to clearly see the denominator and the numerator in each model.*** You could come up with at least 30. Some people will break it down to show 24^{th}s of a carton! You can also, as the learner gains confidence, introduce him/her to "complex fractions" by asking him/her to make a model of fractions like:

$$\frac{2\,\tfrac{3}{4}}{3} \quad \text{and} \quad \frac{2\ 1/3}{4}$$

Note: for this activity, equivalent fractions count as separate models (so, for example, ½ and 6/12 and 2/4 and 3/6 would count as four separate models).

Share your models and your thinking.

Ask learners what they notice, too, about the following:
What happens to the size of the individual parts as the bottom number gets bigger?

8 Ask that the learners simply fill in 8 slots in the carton. Then ask the learners to use the yarn to find different names for this fraction of a carton.

When you all share the fractions you came up with, explain that these are equivalent fractions – all the fractions are equal in the sense that they represent exactly the same quantity.

Do the same filling 10 slots, filling 3 slots, filling 12 slots. Record all the equivalent fractions you come up with for each and discuss your thinking together.

Additional key concepts to pull out of this/to elicit from the learner:

- ***the bigger the denominator, the smaller the size of the individual sub-sections,***
- ***if you have the same numerator and denominator, you have one whole,***
- ***the "reduced fraction" is the one that uses the fewest number of parts to name the fraction.***

9 Ask the learners to build a model of 13/12. Talk about this together. Give them time to think and grapple with it. Talk with the learner about how one carton is still equal to one whole. Do the same with 15/12, 4/2, 7/6, 7/3, etc. Ask the learners what they notice happens when the numerator is bigger than the denominator.

10 Ask learners to build a model of 1 5/12 and to rename it as all 12ths.
Ask learners to build a model of 2½ and to rename this as all halves.
Do the same with some other fractions – really SEEING the mixed numbers and their improper fraction equivalents.

11 Ask learners to make a model of 1/3 and then 3/1, ½ and then 2/1 (to get at how $a/1 = a$, a key idea needed for multiplying and dividing with fractions). Again, let them grapple with it.

12 Now we're going to extend it beyond cartons of a dozen. On the graph paper, sketch several cartons of 16. Determine together some fractions that could be shown on it.

Do the same with a carton of 20, of 7, or whatever. Explore this together, sharing the models you all come up with.

13 From here, do a number of things to explore, in a hands-on way, the meaning of fractions:
A Bring in a couple of dollars in change, with a variety of coins. Ask the learners to take out a dollar and to make a model of 1/5 of a dollar, 4/5 of a dollar, 1/20 of a dollar, 7/20 of a dollar, etc. Ask them to make whatever fractions of a dollar they can think of! Ask them, too, to determine 1 cent as a fraction of a dollar, 10 cents as a fraction of a dollar, etc.

B Do the same with fractions of an hour. Take a big face clock off the wall, if there is one, and use string to make the subdivisions.

C Do the same thing with fractions of a yard or fractions of a circle (360 degrees). Again, use the hands-on objects.

14 Take it to some more abstract fraction situations. (see "Fraction Meaning Situations handout #1"). These situations don't have any questions, but instead require that the learner make mathematical observations. It would be great to tweak these so that they're situations that take into account your learners' lives.

The teacher/tutor does modeling/thinking aloud first based on the situations in Fraction Meaning Situations handout #1. Use a sketch (a rectangle is easy enough to draw or a line that you can divvy up into sections) to represent the fractions. Explain that while the sketches can take a bit of time initially, most people get to the point eventually where they see the fraction or sketch in their mind's eye, and they don't need to actually draw anything.

Then, the learners and tutor both can work independently on sketching situations (see "Fraction Meaning Situations handout #2"). Compare sketches and observations.

15 Make up a bunch of problems that use the same numbers, but reflect different "wholes." Have the learners work these out with a sketch. (Make calculators available). For example:

80 apprentices, or 5/8 of the apprentices on the list, got called out to work. How many apprentices are still waiting to be called out?

There are 80 apprentices on the list. 5/8 of them got called out. How many apprentices are still waiting to be called out?

$1080.72, which is 4/9 of your gross paycheck, went towards your mortgage payment. How much was your gross paycheck?

$1080.72 was your gross paycheck. 4/9 of it went towards your mortgage payment. How much was your mortgage payment?

16 Teach ways to find equivalent fractions through pure number-crunching – but also teach a reason WHY this method works.

Review 2 things that we know:

10 x 1 = 10 and 17 x 1 is 17, etc.
Also, 2 divided by 1 = 2 and 500 divided by 1 = 500, etc.

AND 4/4, 5/5, 7/7, 21/21, 12/12, etc. are all equal to 1.

This gets at one reason why it works to multiply or divide the numerator and denominator of a fraction by the same number in order to end up with an equivalent fraction (because really you're just multiplying by "1"). Relate to equivalent fractions of a dozen that the learners already observed in their models and know to be true.

17 Explore the Division Concept of a Fraction"
Remind learners that we've been looking at the "Part-to-Whole Concept" of a fraction. Ask people once again to explain what the top and bottom numbers mean in this concept.

Using money or a ruler or egg cartons, ask people to make a model of ¾ using the part-to-whole concept of a fraction. Set this model aside.

Then, explain that in the division concept of a fraction, ¾ means 3 divided by 4 or 3 divided into 4 equal parts. Ask learners to make a model of ¾ using the division concept of a fraction. They'll need 3 whole feet, egg cartons or dollars that they'll split into 4 equal parts. Ask them then to take note of what's in 1 of the four parts. Compare it with the first model they made – and they'll notice that both fraction concepts end up representing the same amount in the end!

Do a few different fractions in the same way – modeling the part-to-whole concept and the division concept for the same fraction. With money you could use, for example: 3/5, 2/3, 2/10, etc. With the foot you could do: 5/6, 2/3, 3/12, etc.